5139

FORM TP 2015139

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MAY/JUNE 2015

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

MATHEMATICAL APPLICATIONS

UNIT 2 – Paper 02

2 hours 30 minutes

25 MAY 2015 (p.m.)

This examination paper consists of THREE sections: Discrete Mathematics, Probability and Distributions, and Particle Mechanics.

Each section consists of 2 questions.
The maximum mark for each section is 50.
The maximum mark for this examination is 150.
This examination consists of 7 printed pages and 1 answer sheet for Question 1 (a) (ii) and (iii).

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. Answer ALL questions from the THREE sections.

2. Unless otherwise stated in a question, all numerical answers MUST be given exactly OR correct to three significant figures as appropriate.

Examination Materials

Mathematical formulae and tables (Revised 2010)
Electronic calculator
Ruler and graph paper

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

MODULE 1: DISCRETE MATHEMATICS

Answer BOTH questions.

1. (a) A firm manufactures glass bottles and plastic bottles. All of the bottles have to pass through a sterilizing machine and a capping machine. A case of glass bottles requires 4 minutes on the sterilizing machine and 3 minutes on the capping machine. A case of plastic bottles requires 4 minutes on the sterilizing machine and 6 minutes on the capping machine. Each machine has 50 hours available in a week.

Each week, a profit of $20 per case of glass bottles and $15 per case of plastic bottles can be made.

(i) By identifying the variables, formulate

a) the profit function that needs to be solved **[3 marks]**

b) the inequalities to be used to solve this problem. **[7 marks]**

(ii) **On the graph sheet provided**,

a) draw the graphs of the inequalities made in (i) b) above **[6 marks]**

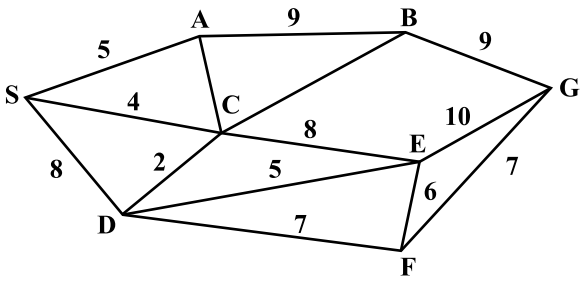
b) identify the feasible region to solve the problem. **[2 marks]**

(iii) From your graph, determine

a) the number of EACH type of bottle required for maximum profit **[3 marks]**

b) the maximum profit. **[2 marks]**

(b) The graph below represents the travel times between Town S and Town G along a number of roads.



Determine the SHORTEST and LONGEST path from Town S to Town G. **[2 marks]**

Total 25 marks

2. (a) In a primary school, four teachers are to be assigned to four different classes, so that each teacher is assigned to only one class. The average time per week, in hours, for each teacher to teach each of the classes is given in the table below.

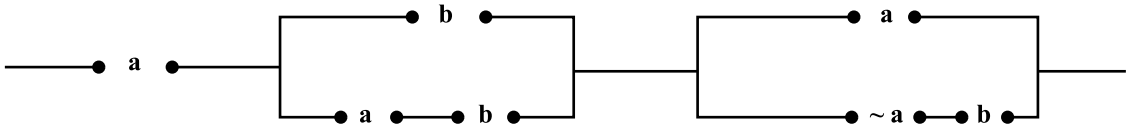
	Class 1	**Class 2**	**Class 3**	**Class 4**
Mrs Jones	30	32	31	34
Mr James	35	33	30	30
Mrs Wright	29	31	28	33
Ms Small	32	34	29	32

(i) Use the Hungarian algorithm to determine the class to which each teacher must be assigned in order to minimize the total time. **[8 marks]**

(ii) Hence, determine the total time spent in the classroom by the four teachers. **[2 marks]**

(b) By constructing a truth table, determine whether $(p \wedge q) \rightarrow p$ is a tautology or a contradiction. **[3 marks]**

(c) (i) Write down a Boolean expression for the following circuit.



[4 marks]

(ii) Simplify the expression obtained in (c) (i) above and hence draw the corresponding circuit. **[8 marks]**

Total 25 marks

SECTION B

MODULE 2: PROBABILITY AND DISTRIBUTIONS

Answer BOTH questions.

3. (a) Twenty per cent of the bolts manufactured by a factory are defective.

(i) Determine the probability that in a random sample of 15 bolts, more than one is defective. **[4 marks]**

(ii) If X represents the number of defective bolts manufactured by the factory, determine the SMALLEST value of n for which the ratio of the standard deviation of X to the mean of X is less than 0.1. **[8 marks]**

(b) If $X \sim Bin(500, 0.005)$, use a suitable approximation to find $P(X \leq 2)$. **[5 marks]**

(c) If $X \sim P_0(20)$, use a suitable approximation to find $P(X \leq 25)$. **[5 marks]**

(d) If X follows a geometric distribution with probability 0.3, determine the probability that X is less than 4. **[3 marks]**

Total 25 marks

4. (a) A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} a + bx & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Given that $F(0.5) = 0.8$,

(i) show that $a = 2.2$ and determine the value of b **[10 marks]**

(ii) determine $P(X > 0.5)$. **[3 marks]**

(b) A random variable X is assumed to have a normal distribution with mean 9 and standard deviation 1. The table below shows the observed and expected values for given ranges of X.

X	$x < 8$	$8 \leq x < 9$	$9 \leq x < 10$	$x \geq 10$	Total
Observed	80	180	50	90	400
Expected	63.5	m	136.5	n	

(i) Determine the value of m and n shown in the table. **[3 marks]**

(ii) Carry out an χ^2 goodness-of-fit test at the 5% significance level to determine whether the data may be modelled by a normal distribution with mean 9 and standard deviation 1. **[9 marks]**

Total 25 marks

SECTION C

MODULE 3: PARTICLE MECHANICS

Answer BOTH questions.

[Take g as 10 ms^{-2}]

5. (a) A particle of mass 5 kg rests on a rough plane surface which is inclined at $\sin^{-1}(\frac{1}{10})$ to the horizontal. A light, inelastic string connects the particle over a weightless, smooth pulley at the top of the surface to another particle of mass 6 kg which hangs freely. The coefficient of friction between the 5 kg mass and the surface is $\frac{1}{4}$.

(i) Draw a force diagram to illustrate this information. **[3 marks]**

(ii) The particle is released from rest. Determine the tension, T, in the string and the acceleration, a, of the particle. **[8 marks]**

(iii) Calculate the force exerted by the string on the pulley. **[4 marks]**

(b) A particle, initially at the origin, moves in the plane so that its velocity at time, t, is given by $\boldsymbol{v} = 5t^2\mathbf{i} - (t - 4)\mathbf{j}$. Find in vector form

(i) the acceleration when $t = 3$ **[2 marks]**

(ii) the position of the particle when $t = 3$. **[4 marks]**

(c) A particle of mass m falls vertically from rest through a medium which has resistance to motion proportional to $\boldsymbol{v}$, where $\boldsymbol{v}$ is the velocity of the particle at time t. Obtain a differential equation relating v and t. **[4 marks]**

Total 25 marks

6. (a) A pressure washer hose is used to clean the wall of a building. The hose delivers 10 kg of water per second as a horizontal stream and hits the wall at a speed of 30 ms^{-1}. Find the average force exerted on the wall, assuming that the water does not bounce off the wall. **[3 marks]**

(b) A five-tonne truck, moving at 4 ms^{-1}, and a three-tonne truck, moving at 7 ms^{-1}, are travelling along the same road in opposite directions. If the trucks collide and become coupled on impact, find the velocity and direction at which they continue to move. **[6 marks]**

(c) A block, A, of mass $3M$ kg is in contact with a smooth table top. A light, inextensible string of length 1.2 metres is attached to one end of the block and passes over a smooth, weightless pulley, which is at the edge of the table. The other end of the string is attached to a block, B, of mass $1.5M$ kg and hangs freely. Initially the system is held at rest with the $3M$ kg mass 0.9 metres from the edge of the table. If the system is released from rest, find, using the principle of conversation of mechanical energy, the speed of block A when it reaches the edge of the table. **[12 marks]**

(d) Consider a string which passes over a fixed, smooth, weightless pulley. If one end of this string is attached to a block with a mass 3 kg, while the other end is attached to another block with a mass of 5 kg, then calculate the acceleration of the system. **[4 marks]**

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.